10/16/16

Review for Q1 Exam 2

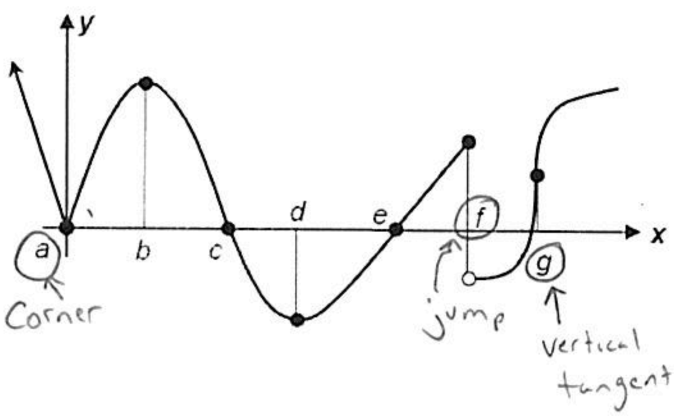
Do Now:

$* \sqrt{x} = x^{\frac{1}{2}}$

Find the derivative: $g(x) = \boxed{(\sin x)}\boxed{(4\sqrt{x})}$

$g'(x) = (\sin x)(2x^{-1/2}) + (4\sqrt{x})(\cos x)$

1. Use the graph below to determine all x-values where the function is **not** differentiable.



$\boxed{a, f, g}$

2. Find the x-coordinates of all points on the graph of $y = x^3 - 5x^2 - 8x + 9$ at which the tangent line is horizontal.

horizontal means a slope of 0

$y' = 3x^2 - 10x - 8$

$0 = 3x^2 - 10x - 8$

$0 = 3x^2 - 12x + 2x - 8$

$0 = 3x(x-4) + 2(x-4)$

$0 = (x-4)(3x+2)$

$x = 4 \quad | \quad x = -\frac{2}{3}$

$\boxed{x = -\frac{2}{3}, 4}$

3. Use the limit definition of a derivative to calculate the derivative of $f(x) = 5 - 2x^2$

$$f'(x) = \lim_{h \to 0} \frac{5 - 2(x+h)^2 - (5 - 2x^2)}{h}$$

$$\frac{5 - 2(x^2 + 2xh + h^2) - 5 + 2x^2}{h}$$

$$\frac{\cancel{5} - \cancel{2x^2} - 4xh - 2h^2 \cancel{- 5} + \cancel{2x^2}}{h}$$

$$\frac{-4xh - 2h^2}{h} \rightarrow -4x - 2h$$

$$\lim_{h \to 0} -4x - 2(0)$$

$$f'(x) = \boxed{-4x}$$

4. Find an equation of the tangent line to the graph of the function $y = 5x^2 - 3x$ when $x = -1$

Point: Use function

$y = 5(-1)^2 - 3(-1)$

$y = 5 + 3$

$y = 8$

$(-1, 8)$

Slope: use derivative

$y' = 10x - 3$

$y' = 10(-1) - 3$

$y' = -13$

$m = -13$

$$\boxed{y - 8 = -13(x + 1)}$$

5. Find $h'(x)$ when $h(x) = 4\sqrt{x} + 5\cos x$

$h(x) = 4x^{1/2} + 5\cos x$

$h'(x) = 2x^{-1/2} + 5(-\sin x)$

$$\boxed{h'(x) = \frac{2}{\sqrt{x}} - 5\sin x}$$

6. Find the h'(x) if $h(x) = \boxed{(\cos(x))}\boxed{(3x^3 - x^2 + 10x + 2)}$

$$h'(x) = (\cos x)(9x^2 - 2x + 10) + (3x^3 - x^2 + 10x + 2)(-\sin x)$$

7. Use the following table to find y' at x = 1, if:

f(1)	f ' (1)	g(1)	g ' (1)
3	4	1	-2

y = f(x)g(x)

$y' = f(x)\cdot g'(x) + g(x)\cdot f'(x)$

$y' = (3)(-2) + (1)(4)$

$y' = -6 + 4$

$\boxed{y' = -2}$

8. Find the coordinates of the point(s) where $f(x) = \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2$ has horizontal tangents. (Derivative = 0)

Need to find the y-values

$f(0) = \frac{1}{4}(0)^4 - \frac{1}{3}(0)^3 - 0^2$

$f(0) = 0$

$f(2) = \frac{1}{4}(2)^4 - \frac{1}{3}(2)^3 - 2^2$

$= 4 - \frac{8}{3} - 4$

$f(2) = -\frac{8}{3}$

$f(-1) = \frac{1}{4}(-1)^4 - \frac{1}{3}(-1)^3 - (-1)^2$

$= \frac{1}{4} + \frac{1}{3} - 1 = \frac{-5}{12}$

$f'(x) = x^3 - x^2 - 2x$

$0 = x^3 - x^2 - 2x$

$0 = x(x^2 - x - 2)$

$0 = x(x-2)(x+1)$

x=0 | x=2 | x=-1

Points

(0,0)

$(2, \frac{-8}{3})$

$(-1, \frac{-5}{12})$

9. Find $f'(x)$ if $f(x)=\boxed{3x^2}\boxed{\sin x}$

$$\boxed{f'(x) = 3x^2(\cos x) + (\sin x)(6x)}$$

Find the equation of the tangent line to the function at the given x-value for each of the following:

10) $f(x)=x^2+8x+16 \quad when\ x=-2$

Point:

$f(-2) = (-2)^2+8(-2)+16$

$f(-2) = 4-16+16$

$f(-2)=4$

$-2,4$

Slope: $f'(x) = 2x+8$

$f'(-2) = 2(-2)+8$

$f'(-2) = -4+8$

$f(-2) = 4$

$m=4$

$$\boxed{y-4=4(x+2)}$$

11) $f(x)=3x^2-4x+2 \quad when\ x=2$

Point:

$f(2)=3(2)^2-4(2)+2$

$f(2) = 12-8+2$

$f(2) = 6$

$(2,6)$

Slope: $f'(x)=6x-4$

$f'(2)=6(2)-4$

$f'(2) = 12-4$

$f'(2)=8$

$m=8$

$$\boxed{y-6=8(x-2)}$$

12) $f(x)=(3x-5)(x^2+9x) \quad when\ x=1$

Point: $f(1) = (3(1)-5)((1)^2+9(1))$

$f(1) = (3-5)(1+9)$

$f(1) = (-2)(10)$

$f(1) = -20$

$(1,-20)$

Slope: $f'(x) = (3x-5)(2x+9)+(x^2+9x)(3)$

$f'(1) = (3(1)-5)(2(1)+9)+(1^2+9(1))(3)$

$= (-2)(11)+(10)(3)$

$= -22+30$

$f'(1) = 8$

$$\boxed{y+20=8(x-1)}$$

13) $f(x)=\sqrt{3x-3} \quad when\ x=4$ (Use the limit definition)

13) $f(x) = \sqrt{3x-3}$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{3(x+h)-3} - \sqrt{3x-3}}{h} \cdot \frac{\sqrt{3(x+h)-3} + \sqrt{3x-3}}{\sqrt{3(x+h)-3} + \sqrt{3x-3}}$$

$$= \lim_{h \to 0} \frac{3(x+h)-3-(3x-3)}{h\left(\sqrt{3(x+h)-3} + \sqrt{3x-3}\right)} \Rightarrow \frac{\cancel{3x}+3h-\cancel{3}-\cancel{3x}+\cancel{3}}{h\left(\sqrt{3(x+h)-3} + \sqrt{3x-3}\right)}$$

$$= \lim_{h \to 0} \frac{3h}{h\left(\sqrt{3(x+h)-3} + \sqrt{3x-3}\right)} \Rightarrow \frac{3}{\sqrt{3(x+h)-3} + \sqrt{3x-3}}$$

$$= \lim_{h \to 0} \frac{3}{\sqrt{3(x+0)-3} + \sqrt{3x-3}} \Rightarrow \frac{3}{\sqrt{3x-3} + \sqrt{3x-3}} = \frac{3}{2\sqrt{3x-3}}$$

Point:

$f(4) = \sqrt{3(4)-3}$

$= \sqrt{12-3}$

$= \sqrt{9}$

$f(4) = 3$

$(4, 3)$

Slope:

$$f'(4) = \frac{3}{2\sqrt{3(4)-3}}$$

$$= \frac{3}{2\sqrt{12-3}} = \frac{3}{2\sqrt{9}}$$

$$= \frac{3}{2 \cdot 3} = \frac{3}{6} = \frac{1}{2}$$

$m = \frac{1}{2}$

$$\boxed{y-3 = \frac{1}{2}(x-4)}$$